DOCUMENT RESUME

ED 421 357	SE 061 613
AUTHOR	Rochowicz, John A., Jr.
TITLE	Sequences, Series, and Spreadsheets: A Mathematical Excursion.
PUB DATE	1998-06-15
NOTE	16p.
PUB TYPE	Guides - Classroom - Teacher (052) -- Reports - Research (143)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Educational Technology; Elementary Secondary Education; Mathematics Activities; *Mathematics Education; Mathematics Instruction; *Spreadsheets; Teaching Methods

ABSTRACT

The use of technology allows students to look at mathematical concepts in many different ways. With a variety of perspectives, studying ideas that were at one time difficult to understand are possible. Mathematics learning focuses more on concepts and less on computations. Various sequences including arithmetic, geometric, and partial sum can be studied in explicitly defined and recursive forms. This paper describes spreadsheet macros for automating computations and graphing relationships for these sequences. Learning mathematics with technology enables the student to develop intuition for correct results and to do meaningful mathematics. Classroom examples illustrate the relevance of the mathematics concepts studied. As technology continues to be part of many facets of society, studying mathematics with technology is also becoming increasingly necessary. (Author)

Sequences, Series,

and Spreadsheets:

A Mathematical Excursion

John A. Rochowicz, Jr.

Alvernia College

Department of Mathematics

Abstract

The use of technology allows students to look at mathematical concepts in many different ways. With a variety of perspectives, studying ideas that were at one time difficult to understand are possible. Mathematics learning focuses more on concepts and less on computations.

Various sequences including arithmetic, geometric, and partial sum can be studied in explicitly defined and recursive forms. This paper describes spreadsheet macros for automating computations and graphing relationships for these sequences. Learning mathematics with technology enables the student to develop intuition for correct results and to do meaningful mathematics.

Classroom examples illustrate the relevance of the mathematics concepts studied. As technology continues to be part of many facets of society, studying mathematics with technology is also becoming increasingly necessary.

Introduction

The use of technology is driving reform in many facets of mathematics education (Willoughby, 1990, Steen, 1989). Due to poor preparation, anxiety, and lack of motivation displayed by many students, meaningful learning of mathematics is very difficult to achieve.

With the use of technology, the potential exists for the learning of mathemtics to be more motivational and active. When any field of study, especially mathematics is motivational, learning increases and improves (Keller & Suzuki, 1988).

Although learning to use technology may interfere with the learning of mathematics, relevant situations can be explored in more visual and interactive ways. This paper describes how to calculate the terms of a sequence and how to apply these calculations to practical problems with the use of the LOTUS 1-2-3 spreadsheet.

Although there is more sophisticated software available such as MATHEMATICA, MAPLE or DERIVE, the use of a spreadsheet provides the student with the capability to do and see calculations. The learner interacts with mathematical concepts as he does with pencil and paper. With a transparent and easy to use interface, spreadsheet software is practical for learning mathematics.

Sequences

An explicitly defined or iterative sequence calculates the terms of a sequence by knowing the general or nth term of the specific sequence. For example, consider the sequence of numbers 1,3,5,7,9... For n = 1 the first term is $a_1 = 2(1)-1 = 1$ and the general or nth term is 2_n-1.

Recursively defined sequences are sequences that are calculated by knowing the first few terms and then computing the succeeding terms by using the previous terms. The terms of this sequence can also be found recursively. Let the first term be represented by $a_1 = 1$, then the next terms in the sequence are found by adding 2 to the previous term. That is for the next term, $a_2 = a_1 + 2$, and in general, $a_{n+1} = a_n + 2$. Many sequences can be described in both explicitly defined form and recursive form.

Sequences are used to model population growth, depreciation, compound interest, annuities, drug dosages, spread of diseases, spread of information, inflation, genetics, and many more situations. Swokowski (1994) and Piascik (1992) describe a number of applications of arithmetic and geometric sequences. Larson, Hostetler, and Edwards (1998) define partial sum sequences and present a variety of applications. Rogers, Haney and Laird (1993) provide many examples for business mathematics such as annuities, compound interest and depreciation.

Defining the Macros

O'Leary and O'Leary (1990) describe how to construct spreadsheet macros for DOS. Hayen (1994) shows how to develop macros for WINDOWS.

Macros that calculate the terms of sequences in iterative (explicitly defined) form and recursive form are provided in Table 1 and Table 2. Below Table 1 and Table 2 are descriptions of what each macro does. Although, the starting cell for the calculations can be any cell, the tables below show cell a11 as the starting position. In this way, space at the top of the spreadsheet is provided to allow the user to title or document the problem being studied. The use of a "$" symbol around cell letters keeps specified cell values fixed. The slash "/" with a group of letters is used to perform a desired task. For example, /gtxx{?}~ makes a graph of type xy and waits for the user to highlight the data for column x.

Table 1.

The Iteration Macro

```
(1) {goto}a11~{?}~{goto}a12~+a11+1~

(2) /c~a12.a70~

(3) {goto}c11~{?}~/c~c12.c70~

(4) {goto}d11~@sum($c$11.c11)~/c~d12.d70~

(5) /gtxx{?}~a{?}~vq
```

Note. Using the TOOLS MACRO RUN command sequence from the top menu runs the macro starting at the indicated range on a spreadsheet. If a series or sequence of partial sums is desired use line 4. If graphing the relationship is desired use line 5. Copying cell formulas to a70, c70, and d70 can be adjusted as needed.

The iteration macro does the following:

Line 1 asks the user to enter the starting value for n in cell a11, goes to a12 and adds 1 to the value in a11.

Line 2 copies the formula in a12 down to a70.

Line 3 requests the user to input the nth term of the sequence in cell c11 then copies nth term cell formula down to c70. (Line 3 calculates 60 terms of the sequence.)

Line 4 calculates the sequence of partial sums for each iteration down to d70.

Line 5 allows the user to graph the terms of the sequence versus the values of n or the terms of the partial sum sequence versus the values of n.

Table 2.

The Recursion Macro

---

(1) {goto}a11~{?}~{goto}a12~+a11+1~

(2) /c~a12.a70~

(3) {goto}c11~{?}~

(4) {goto}c12~{?}~/c~c12.c70~

(5) /gtxx{?}~a{?}~vq

Note. Using the TOOLS MACRO RUN command sequence from the top menu runs the macro starting at the indicated range on a spreadsheet. If graphing is desired apply line 5, if not take out line 5. Copying formulas to a70 and c70 can be adjusted as needed.

The recursion macro does the following:

Line 1 asks the user to enter starting value for n goes to a12 and adds 1 to the value in a11.

Line 2 copies the formula in a12 down to a70.

Line 3 waits for user to input first term of sequence in c11.

Line 4 waits for user to enter the recursive form of the sequence in c12 and copies this formula down to c70.

Line 5 graphs the terms of the sequence versus the values of n or the terms of the partial sum sequence versus the values of n.

Classroom Examples

Relevant situations illustrate the usefulness of mathematics. The student should be able to calculate results through iteration, recursion and by a formula that defines the situation. Assignments can be considered for group and individual projects. The next two examples are typical of what I use for my classroom.

Example 1. A medicine containing 1050 mg of a chemical compound is given to a patient. If the drug is eliminated from the body at the rate of 30% each hour, find the number of milligrams of the drug remaining in the bloodstream after 6 hours.

Example 2. An automobile company produces cars at a fixed production cost of $500,000 and an additional cost of $24,000 for each car. Find the cost to produce 50 cars.

Solutions to the Examples

Each example illustrates concepts that can be solved in different ways. The solution to Example 1 can be obtained by using $S = P(1 - i)^n$, where P represents the starting amount, i the rate of drug elimination and n the time in hours. That is, $S = 1050 * (1 - 0.3)^6 = 123.53$ mg.

The spreadsheet macros defined in Table 1 and Table 2 determine the result by iteration and recursion. The input for the iteration macro is: enter 0 in cell a11 and enter 1050 * (0.7)^a11 in cell c11. The macro does the rest. The input for the recursion macro is: enter 0 in cell a11, enter 1050 (initial dose) in c11, and enter 0.7 * c11 in c12. The result from either macro is 123.53 mg. Visualizing this relationship is achieved by graphing column A for x and column C for y.

Table 3.

Iteration Output and Cell Formulas for Example 1

	A	C	A	C
	n	terms		
11	0	1050	0	1050*0.7^a11
12	1	735	+a11+1	1050*0.7^a12
13	2	514.5	+a12+1	1050*0.7^a13
14	3	360.15	+a13+1	1050*0.7^a14
15	4	252.11	+a14+1	1050*0.7^a15
16	5	176.47	+a15+1	1050*0.7^a16
17	6	123.53	+a16+1	1050*0.7^a17
18	7	86.47	+a17+1	1050*0.7^a18
19	8	60.53	+a18+1	1050*0.7^a19
20	9	42.37	+a19+1	1050*0.7^a20
21	10	29.66	+a20+1	1050*0.7^a21
22	11	20.76	+a21+1	1050*0.7^a22
23	12	14.53	+a22+1	1050*0.7^a23
24	13	10.17	+a23+1	1050*0.7^a24
25	14	7.12	+a24+1	1050*0.7^a25
26	15	4.98	+a25+1	1050*0.7^a26

Note. Cell formulas are displayed in the second group of cells A and C.

Table 4.

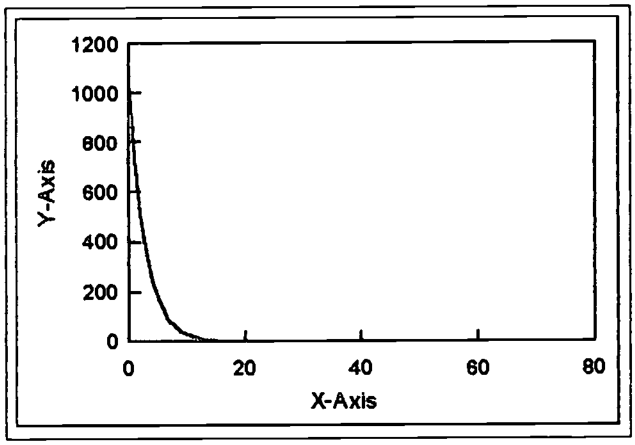
Recursion Output and Cell Formulas for Example 1

	A	C	A	C
	n	terms		
11	0	1050	0	1050
12	1	735	+a11+1	0.7*c11
13	2	514.5	+a12+1	0.7*c12
14	3	360.15	+a13+1	0.7*c13
15	4	252.11	+a14+1	0.7*c14
16	5	176.47	+a15+1	0.7*c15
17	6	123.53	+a16+1	0.7*c16
18	7	86.47	+a17+1	0.7*c17
19	8	60.53	+a18+1	0.7*c18
20	9	42.37	+a19+1	0.7*c19
21	10	29.66	+a20+1	0.7*c20
22	11	20.76	+a21+1	0.7*c21
23	12	14.53	+a22+1	0.7*c22
24	13	10.17	+a23+1	0.7*c23
25	14	7.12	+a24+1	0.7*c24
26	15	4.98	+a25+1	0.7*c25

Note. Cell formulas are displayed in the second group of cells A and C.

Graph 1

Visualizing Example 1.



In Table 3 and Table 4 output is displayed for Example 1 by iteration and recursion. Graph 1 shows the relationship visually.

Solutions to Example 2 can also be achieved in different ways. Students apply the concepts of a linear cost function where the fixed cost is \$500,000 and production cost is \$24000 for each car. The linear cost function is given by C(x) = 500,000 + 24,000x where x is the number of cars produced. When x is 50 cars the cost is 500,000 + 24,000(50) = \$1,700,000.

When using the iteration macro enter 0 in cell a11 and enter 500000 + 24000*a11 in cell c11. Press ENTER. The macro calculates the output and graphs the cost versus the number of cars. Notice that for x is 50 the corresponding output is 1700000. The graph is a straight line going up.

When using the recursion macro, enter 0 in cell a11 and enter 500000 in cell c11. Then enter \$c\$11+24000*a12 in c12 and press ENTER (or enter +c11+24000 in c12). As before, the macro calculates and graphs the relationship. The graph obtained is the same and the output is 1700000. To the student different ways of getting the correct result reinforces knowledge and builds confidence in doing mathematics.

Table 5

Iteration Output and Cell Formulas for Example 2

	A	C	A	C
	n	terms		
11	0	500000	0	50000+24000*a11
12	1	524000	+a11+1	500000+24000*a12
13	2	548000	+a12+1	500000+24000*a13
14	3	572000	+a13+1	500000+24000*a14
15	4	596000	+a14+1	500000+24000*a15
16	5	620000	+a15+1	500000+24000*a16
17	6	644000	+a16+1	500000+24000*a17
18	7	668000	+a17+1	500000+24000*a18
60	49	1676000	+a59+1	500000+24000*a60
61	50	1700000	+a60+1	500000+24000*a61
62	51	1724000	+a61+1	500000+24000*a62

Note. Cell formulas are displayed in the second group of cells A and C. The iteration macro calculates output to cells a70 and c70.

Table 6.

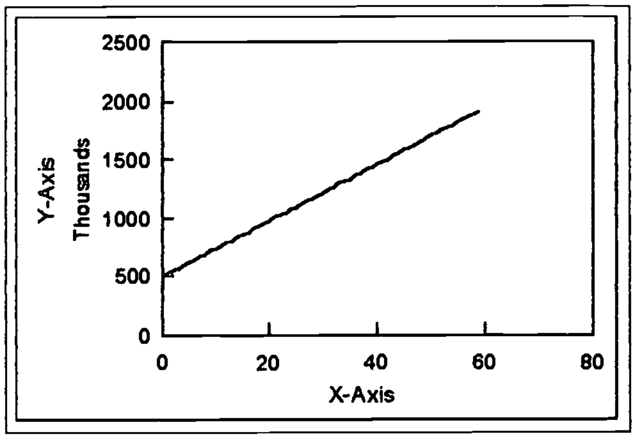
Recursion Output and Cell Formulas for Example 2

	A	C	A	C
	n	terms		
11	0	500000	0	500000
12	1	524000	+a11+1	c11+24000
13	2	548000	+a12+1	c12+24000
14	3	572000	+a13+1	c13+24000
15	4	596000	+a14+1	c14+24000
16	5	620000	+a15+1	c15+24000
17	6	644000	+a16+1	c16+24000
18	7	668000	+a17+1	c17+24000
60	49	1676000	+a59+1	c59+24000
61	50	1700000	+a60+1	c60+24000
62	51	1724000	+a61+1	c61+24000

Note. Cell formulas are displayed in the second group of cells A and C. The recursion macro calculates output to cells a70 and c70.

Graph 2

<u>Visualizing Example 2.</u>



In Table 5 and Table 6, output is displayed for Example 2 by iteration and recursion. Graph 2 shows the relationship visually.

Classroom Experiences

I have used these macros for my business mathematics and precalculus mathematics courses. Topics such as compound interest, ordinary annuities, amortization of loans, drug effectiveness, and half-life of radioactive isotopes are studied. Visualizing the ouput in Example 1 shows the student the meaning of decay. In Example 2 the concept of linear growth is displayed.

From my classroom experiences I have learned a lot about technology use for learning mathematics. When technology is used for studying sequences, students see more connections among iteration, recursion, and defined equations. Demonstrating knowledge to novel situations improves for many students. Practical exercises motivate learning. Students increase skills in interpretation of input and output, and in analysis and formulation of problems in mathematical terms. Meanings of terms such as recursion, iteration, convergence, and divergence make sense to many more students. Clearly activity increases in doing and learning mathematics.

Conclusions

The terms of sequences can be calculated easily with spreadsheets. Learning to use a spreadsheet does not interfere with learning mathematics. Graphing capabilities makes this software useful and beneficial to the learning experience. A variety of approaches to calculating different sequences can be studied. Many students appear to be more active in their learning; to be in tune with the material being discussed; and to enjoy the relevance of the concepts being presented. Visualizing concepts is also helpful in studying mathematics.

As society continues evolving technologically, learning and doing mathematics must apply more technological aspects. Technology use allows mathematics learning to be a problem solving endeavor of meaningful and relevant ideas. Making connections among ideas, improving motivation, and thinking about mathematics are beneficial for student learning.

References

Hayen, Roger. (1994). Introductory LOTUS 1-2-3 Release 4 for WINDOWS. Cambridge, MA: Course Technology, Inc.

Keller, J. M., & Suzuki, K. (1988). Use of the ARCS Motivation Model in Courseware Design. In D. H. Jonassen (Ed.), Instructional Designs for Microcomputer Courseware Hillsdale, NJ: Lawrence Earlbaum Associates.

Larson, R. E., Hostetler, R. P. & Edwards, B. H. (1998). Calculus With Analytic Geometry. (Sixth Edition). Lexington, MA: DC Heath and Company.

O'Leary, T. J. & O'Leary L. I. (1990). The Student Edition of LOTUS 1-2-3 Release 2.2. Reading, MA: Addison-Wesley Publishing Company.

Piascik, C. (1992). Applied Finite Mathematics for Business and the Social and Natural Sciences. St. Paul, MN: West Publishing Company.

Rogers, J. E., Haney, B. F. & Laird, D. (1993). Fundamentals of Business Mathematics. Boston, MA: PWS-Kent Publishing Company.

Steen, L. A. (1989). Reshaping College Mathematics. (MAA Notes No. 13). Washington, DC: Mathematical Association of America.

Swokowski, E. W. (1994). Precalculus (Seventh Edition). Boston, MA: PWS-Kent Publishing Company.

Willoughby, S. S. (1990). Mathematics for a Changing World. Alexandria, VA: Association for Supervision and Curriculum Development.

SE061613

ERIC

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
Educational Resources Information Center (ERIC)

REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: Sequences, Series, and Spreadsheets: A mathematical Excursion

Author(s): John A. Rochowicz Jr

Corporate Source:

Publication Date: 6/15/98

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic/optical media, and sold through the ERIC Document Reproduction Service (EDRS) or other ERIC vendors. Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following two options and sign at the bottom of the page.

	The sample sticker shown below will be affixed to all Level 1 documents	The sample sticker shown below will be affixed to all Level 2 documents	
[X] Check here For Level 1 Release: Permitting reproduction in microfiche (4" x 6" film) or other ERIC archival media (e.g., electronic or optical) *and* paper copy.	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY ____ Sample ____ TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN OTHER THAN PAPER COPY HAS BEEN GRANTED BY ____ Sample ____ TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)	[ ] Check here For Level 2 Release: Permitting reproduction in microfiche (4" x 6" film) or other ERIC archival media (e.g., electronic or optical), but *not* in paper copy.
	Level 1	Level 2	

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but neither box is checked, documents will be processed at Level 1.

"I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic/optical media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries."

Sign here→ please

Signature: John A. Rochowicz Jr	Printed Name/Position/Title: John A. Rochowicz, Jr. Ed.D. Professor of Mathematics	
Organization/Address: Alvernia College 400 St. Bernardine St. Reading, PA. 19607-1799	Telephone: 610-796-8257	FAX:
	E-Mail Address:	Date: 6/15/98

ERIC

(over)